NAG Fortran Library Routine Document

F06TRF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F06TRF transforms an n by n complex upper Hessenberg matrix H to upper triangular form R by applying a unitary matrix P from the left or the right. H is assumed to have real non-zero sub-diagonal elements $h_{k+1,k}$ for $k = k_1, k_1+1, \ldots, k_2-1$ only; R has real diagonal elements. P is formed as a sequence of plane rotations in planes k_1 to k_2.

If SIDE = 'L', the rotations are applied from the left:

$$PH = R, \quad \text{where} \quad P = DP_{k_2-1} \cdots P_{k_1+1}P_{k_1},$$

and $D = \text{diag}\big(1, \ldots, 1, d_{k_2}, 1, \ldots, 1\big)$ with $\big|d_{k_2}\big| = 1$.

If SIDE = 'R', the rotations are applied from the right:

$$HP^H = R, \quad \text{where} \quad P = DP_{k_1}P_{k_1+1} \cdots P_{k_2-1},$$

and $D = \text{diag}\big(1, \ldots, 1, d_{k_1}, 1, \ldots, 1\big)$ with $\big|d_{k_1}\big| = 1$.

In either case, P_k is a rotation in the $(k, k+1)$ plane, chosen to annihilate $h_{k+1,k}$.

The 2 by 2 plane rotation part of P_k has the form

$$\begin{pmatrix} \bar{c}_k & s_k \\ -s_k & c_k \end{pmatrix}$$

with s_k real.

2 Specification

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SUBROUTINE F06TRF (SIDE, N, K1, K2, C, S, A, LDA)
INTEGER            N, K1, K2, LDA
double precision   S(*)
complex*16         C(*), A(LDA,*)
CHARACTER*1        SIDE
```

3 Description

None.

4 References

None.

5 Parameters

1: SIDE – CHARACTER*1 *Input*

On entry: specifies whether H is operated on from the left or the right, as follows:

if SIDE = 'L', H is pre-multiplied from the left;
if SIDE = 'R', H is post-multiplied from the right.

Constraint: SIDE = 'L' or 'R'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix H.

Constraint: $\mathrm{N} \geq 0$.

3: K1 – INTEGER *Input*
4: K2 – INTEGER *Input*

On entry: the values k_1 and k_2.

5: C($*$) – ***complex*16*** array *Output*

On exit: $\mathrm{C}(k)$ holds c_k, the cosine of the rotation P_k, for $k = k_1, \ldots, k_2 - 1$; $\mathrm{C}(k_2)$ holds d_{k_2}, the k_2th diagonal element of D, if SIDE = 'L', or d_{k_1}, the k_1th diagonal element of D, if SIDE = 'R'.

6: S($*$) – ***double precision*** array *Input/Output*

On entry: the non-zero sub-diagonal elements of H: $\mathrm{S}(k)$ must hold $h_{k+1,k}$, for $k = k_1, k_1 + 1, \ldots, k_2 - 1$.

On exit: $\mathrm{S}(k)$ holds s_k, the sine of the rotation P_k, for $k = k_1, \ldots, k_2 - 1$.

7: A(LDA,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the upper triangular part of the n by n upper Hessenberg matrix H.

On exit: the upper triangular matrix R. The imaginary parts of the diagonal elements are set to zero.

8: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F06TRF is called.

Constraint: $\mathrm{LDA} \geq \max(1, \mathrm{N})$.

6 Error Indicators and Warnings

None.